Problem solving with an Advanced Computing Environment to learn Mathematics

Cecilia FISSORE, Francesco FLORIS, Marina MARCHISIO, Marta PULVIRENTI

Università degli Studi di Torino, Torino (TO)

Abstract

By solving contextualized problems, students can gain tools to investigate and explain phenomena of the world around us, favoring the development of a conscious citizenship. In our constantly evolving modern society, it is essential for students to acquire digital skills. Therefore, it is even more important to solve problems through technologies, such as an ACE (Advanced Computing Environment). This allows to perform numerical and symbolic computations, to create 2 and 3 dimensions graphs, to write procedures and to program interactive components in order to generalize the resolution. In this paper, the workshop "Problem Solving with an ACE" is presented, which consisted of 4 two-hour long meetings, and involved twenty-four 11th grade students of the upper secondary school "Galileo Ferraris" of Turin. It took place in a computer lab, during curricular hours. Disciplinary topics treated were agreed with the Mathematics teacher of the class, allowing the insertion of the workshop hours in the normal lesson planning. The goal of this paper is to discuss an example of a possible design of problem solving activities with the use of an ACE to teach Mathematics, which could be proposed to students throughout the school year.

Keywords: Problem Solving, Mathematics, Advanced Computing Environment.

Introduction

According to the document "National indications and new scenarios" (MIUR, 2018), the teaching of Mathematics must provide tools to investigate and explain the phenomena of the world around us, favoring a rational approach to problems that reality poses and providing an important contribution to the construction of a conscious citizenship. It assumes that, instead of acquiring a mastery in Mathematics competences, the focus should shift on processes of developing and putting into practice the mathematical thinking to solve a range of problems in everyday situations. Nowadays, problem solving should be, therefore, at the center of mathematical curricula, both as the main objective of education and as the main activity of teaching and learning Mathematics. By learning to solve math problems, in groups or individually, students can acquire ways of thinking, creativity, curiosity, collaborative learning skills and confidence in unfamiliar situations (Leong & Janjaruporn, 2015). At the same time, in our modern society increasingly characterized by the use of ICT (Information and Communication Technologies) it is essential that students acquire digital skills. The use of ICT at school, which increases the interest and motivation of students (Fondazione Giovanni Agnelli, 2010), has to be accompanied by a renewal of teaching that makes the most of its cognitive potential. There are multiple empirical confirmations that. Technologies offer multiple possibilities for the representation and exploration of mathematical tasks: they favor conjectures, justifications and generalizations and allow calculations, data collection and analysis and exploration with multiple representative forms (Goos, Galbraith, Renshaw, & Geiger, 2003). One of the technologies used for problem solving activities is an Advanced Computing Environment (ACE): it allows performing numerical and symbolic calculations, static and animated graphical representations to be created in 2 and 3 dimensions, writing procedures in simple language, programming and connecting all different representation registers in a single worksheet, also using verbal language. The use of an ACE in problem solving can support students in reasoning processes, in the formulation of solution strategies and in the generalization of solutions (Barana & Marchisio, 2017). The technologies, in particular virtual learning environments, also give the possibility to design and structure learning environments in which students can continuously interact with each other, synchronously or asynchronously, to solve problems and develop knowledge and skills (Barana, Brancaccio, et al., 2017; Barana & Marchisio, 2016, 2017). In this article, we present a workshop entitled "Problem Solving with the use of an ACE" of 4 two-hours long meetings, which involved twenty-four 11th grade students from the upper secondary school "Galileo Ferraris" of Turin.

It took place in a computer lab during school hours. The disciplinary topics treated were agreed with the Mathematics teacher of the class, which allowed us to insert our workshop during the regular lessons plan. The objective of the article is to present an example of possible design of problem solving activities with the use of an ACE for learning Mathematics, which could be proposed to students throughout the school year.

State of the art

The term "problem solving" refers to mathematical tasks which provide intellectual challenges that improve students' understanding and mathematical development (National Council of Teachers of Mathematics, 2000). It represents one of the main objectives of the teaching of Mathematics and it consists of the following capabilities: to understand the problem, to develop a mathematical model, to develop the resolution process and to interpret the obtained solution (Samo, Darhim, & Kartasasmita, 2017). In problem solving, it is important to use problems that are contextualized in everyday life to activate in students the modeling skills and teach them to recognize how and when to use their knowledge and to solve problems in real world situations (Baroni & Bonotto, 2015; Samo et al., 2017). Moreover, challenging problems should be used, whose content topics have been studied in class, or will soon be, with open data in order to offer students a vast range of possibilities to choose and make decisions, and that suggest more than one solving strategy. By solving a problem, you can learn new mathematical concepts or reinforce already acquired knowledge. Furthermore, the resolution of a problem by students can be used to assess progress in problem solving skills, using an evaluation rubric with a score scale (Leong & Janjaruporn, 2015). In order to gain problem-solving experience, students should solve different types of problems, individually or in groups, at regular intervals and for an extended period of time (Leong & Janjaruporn, 2015). Solving problems in groups, students can learn how to expose and support their idea (against their peers and the teacher who guides the discussion), learn new concepts and develop collaborative learning skills. Finally, the problem solving activity can also be designed remotely through a virtual learning environment (Barana & Marchisio, 2017). In this case, students can solve a problem individually while exchanging views with other students via a chat or a forum. Analysis of ways in which mathematical problems and the process involved in problem solving are formulated generates important information to structure learning environments in order to guide the construction of students' mathematical concepts (Liljedahl, Santos- Trigo, Malaspina, & Bruder, 2016). An ACE allows students to face a problematic situation in the way that best suits their thinking; it allows them to use on the same page different types of representations based on the chosen strategy and to visualize the whole reasoning together with the verbal explanation. It can therefore meet all the processes involved in the resolution of a problem (Barana, Fioravera, & Marchisio, 2017). Starting from a mental thought, the resolution process can be performed using different registers (words, graphs, numeric and symbolic calculations, etc.) and it can be experienced through computer simulations. Being able to correctly combine quests and methods is a crucial aspect in solving problems, and the fact that they can be used simultaneously in a single environment promotes high levels of clarity and understanding. One of the most obvious features of an ACE is that it helps manage calculations that are difficult to perform by hand. This has a twofold effect: students can concentrate on solving the problem rather than on the calculation, and it expands the variety of problems that can be proposed. Another very important aspect of an ACE is the designing and programming of interactive components that allows showing how results vary depending on the input parameters, thus enabling the generalization of the resolution process. Generalizing is an important process through which the characteristics of a solution are examined and the reasons why it worked are studied. This process is equivalent to a phase of testing, elaborating, inventing and creativity (Liljedahl et al., 2016).

Methodology

The workshop was conducted from three trainers of the University of Turin, selected as tutors within the "Problem Posing and Solving" project (Barana et al., 2018) of the "Direzione Generale per gli Ordinamenti Scolastici e la Valutazione del Sistema Nazionale di Istruzione" ("General Management to the School Systems and Evaluation of the National Education System"). It took place between January and February of the school year 2018-2019, during four two-hour long meetings. Twenty-four 11^{th} grade students from the upper secondary school “Galileo Ferraris” of Turin were involved. The workshop took place in a computer lab equipped with an Advanced Computing Environment, Maple (https://www.maplesoft.com/). The objective of the workshop was to show students how an ACE can help us in solving a problem, without them being required to know how to use the software as a prerequisite, but teaching them how to use it by solving a contextualized problem of Mathematics. In this way, the ACE is not only a tool, but it becomes an effective methodology that can support problem solving and learning Mathematics. The topics of the four meetings were: introduction to the use of an ACE and main commands for algebraic calculation and for graphic and animated representations; generalization of the resolution process with the programming of interactive components; introduction to programming (if/then/else, procedures and cycles); support of an ACE in solving and generalizing a problem. All the topics of the workshop were explained to students through the solving of a problem, a different one for each meeting. Together with the Mathematics teacher, we decided the disciplinary issues involved in the resolution of problems, and as a result, all activities presented to students were aligned with the Mathematics program done in class. Disciplinary topics were respectively: analytic geometry (points and parabola in the Cartesian plane); functions in one variable (domain, sign, zeros) and graphical search for the maximum of a quadratic function; recursive functions, simple exponential growth models; summary of topics seen in previous meetings. All problems used during the meetings were contextualized: the construction of an artistic fountain, the search for the maximum profit for the online sale of a product, the invention of the game of chess and the search for the trajectory of a catapult to sink enemy ships. In all meetings, the generalization of the problem was requested through the creation of interactive components (sliders, text areas, buttons, graphic windows) that can perform any operation and replace different types of output. The working methods were the following: brief frontal explanations, given by trainers, of the presentation of the workshop program and the ACE; collaborative resolution between trainers and students of a contextualized problem, in which students proposed a problem resolution strategy and tutors showed students how to use the ACE to implement the strategy; resolution of a problem by students divided into groups with support and explanations from tutors. During the collaborative resolution between trainers and students, a trainer read the text of the problem and asked students questions to guide them gradually through the resolution. Students proposed a solving strategy, as a result of proposals of one or more students and a brief group discussion. The trainer then showed students, each of them working on a PC, how to use the ACE to implement the proposed resolution strategy and students in turn tried to carry out the procedure on their computer, asking the second trainer for help in case of need. During the last meeting of the workshop, students were divided into 8 groups of 3 students and they were asked to solve a problem using an ACE independently. Two trainers were available for students to help them develop their resolution and to understand the necessary commands. Students could also use the software Help, integrated into the program, which consists of a guide containing all the information on a specific topic or command. For each command, there are many examples that help the student understand its functioning and adapt it to its purpose. At the end of the workshop, each student completed a satisfaction questionnaire. In the problem that students solved in groups, they had to find out if, during a siege, catapults would succeed in destroying the enemy ships. Knowing that catapults can shoot a rock at a rate of approximately 70 *m/s*, that the city walls are located 50 meters above sea level and that the port is 500 meters away, as the crow flies, from the top of walls. They were also asked to draw the launch trajectory needed to hit ships and to represent how the trajectory varies when changing the angle of inclination of the catapult. Eight different resolutions of the problem were produced, one for each group of students. To evaluate how students related with the use of an ACE for the resolution of the problem and their satisfaction with the workshop, we analyzed their solutions and their answers in the final satisfaction questionnaire.

Results and discussion

The problem chosen allowed us to have students think about advantages of using an ACE to solve the problem. In fact, the first request of the problem required the resolution of a linear system of two equations in two unknowns represented by equations of a parabolic motion, in which unknowns were the time and the angle of inclination of the catapult. Students have solved the system mainly using two different resolution strategies. One of these strategies (Fig.1), consisted in defining all data of the problem, and then defining two variables (*x, y*) having equations of motion as values. Then students solved the system by imposing two variables equal to the abscissa and final ordinate of the motion.

$$y := -\frac{1}{2}\cdot g\cdot t^2 + v\cdot \sin(\theta)\cdot t + 50$$

$$x := v\cdot \cos(\theta)\cdot t$$

$$solve\left(\left[y=0, x=\sqrt{500^2-50^2}\right]\right)$$

$$\{t=8.097113026, \theta=0.4996990567\}, \{t=-8.097113026, \theta=-2.641893597\},$$
$$\{t=12.58927705, \theta=0.9709298490\}, \{t=-12.58927705, \theta=-2.170662805\}$$

Figure 1 – Commands related to the first type of solution strategy.

In the second typology (Fig. 2) of solution strategy, students used the substitution method, isolating the variable (*t*) in the first equation and replacing it in the second. Then they replaced the problem data in equations and solved the equation *y=0*.

$$isolate(x_0 + v\cdot\cos(\theta)\cdot t = x, t)$$

$$t = \frac{x-x_0}{v\cos(\theta)}$$

$$eval\left(-\frac{1}{2}\cdot g\cdot t^2 + v\cdot\sin(\theta)\cdot t + y_0, t = \frac{x-x_0}{v\cos(\theta)}\right)$$

$$-\frac{g\,(x-x_0)^2}{2\,v^2\cos(\theta)^2} + \frac{\sin(\theta)\,(x-x_0)}{\cos(\theta)} + y_0$$

$$subs\left(g=9.81, x_0=0, y_0=50, v=70, x=500, -\frac{g\,(x-x_0)^2}{2\,v^2\cos(\theta)^2} + \frac{\sin(\theta)\,(x-x_0)}{\cos(\theta)} + y_0\right)$$

$$-\frac{250.2551020}{\cos(\theta)^2} + \frac{500\sin(\theta)}{\cos(\theta)} + 50$$

$$solve\left(0 = -\frac{250.2551020}{\cos(\theta)^2} + \frac{500\sin(\theta)}{\cos(\theta)} + 50\right)$$

$$0.9650042234, 0.5061234509, -2.176588430, -2.635469203$$

Figure 2 – Commands related to the second type of solution strategy.

The system has four solutions (with angles expressed in radians) but, taking into account the contextualization of the problem, only angles between 0 and 90 degrees can be considered. All students found two angles of inclination that could hit enemy galleys and they represented them graphically. Even in the generalization of the resolution process students reasoned in different ways using interactive components: we show two representative examples. In the first example (Fig. 3), students used two sliders to generalize the angle of inclination of the catapult and the launch speed. By varying two sliders, the representation of the trajectory of the catapult varies in a continuous way.

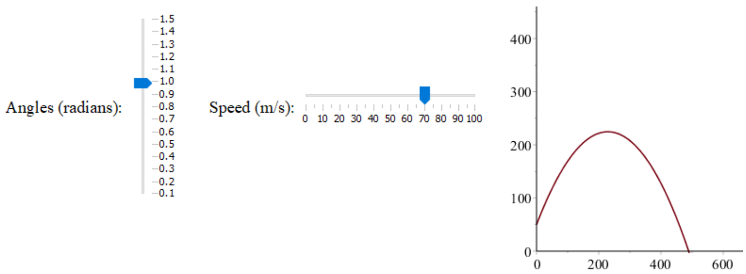

Figure 3 – First example of generalization of the resolution process.

In the second example (Fig. 4) the user can choose, by entering it in the text box, the numerical value of the speed, the height of walls, the distance between walls and enemy ships and the time of launch. Then, by clicking on the "Calculate angle" button, the value of the catapult inclination angle used for the launch is automatically calculated.

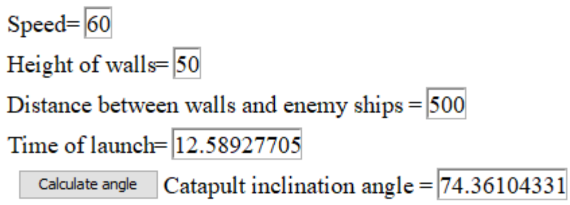

Figure 4 – Second example of generalization of the resolution process.

From the final satisfaction questionnaire, it emerged that overall students really liked the workshop, that was different from usual Mathematics lessons, and all students would like it to be repeated in the future. In the table 1, we focus on students' considerations on various aspects of the use of the ACE for problem solving (expressed with a value from 1, i.e. strongly disagree, to 5, i.e. strongly agree).

Question	*Mean*	*Standard Deviation*
Maple allowed me to focus on reasoning without thinking about calculations	3.84	0.75
Variables and procedures allowed me to speed up the resolution process	3.76	1.09
Graphics allowed me to find the solution of the problem or to check its accuracy	4.08	1.04
Animated graphs allowed me to represent multiple possible solutions	4.20	0.91
Combining text, graphs and formulas helped me in understanding and solving	3.84	0.94
Interactive components allowed me to see how solution changes when data vary	4.64	0.70
Programming interactive components allowed me to understand how to generalize the resolution of a problem	4.20	0.87

Table 1 – Results of questions concerning the ACE

In the final questionnaire, we also asked students which benefits they thought there may be using an ACE to solve a problem rather than using pen and paper or other digital tools. Students said that the program helps in calculations, in concentrating on the process and in visualizing graphs; with the ACE, it is sufficient to build the process to reach all possible solutions, which is not possible using a pen and paper; the ACE limits the calculation errors and inaccuracies in graphs, allowing focusing on the resolution. To the question: "What do you think you have learned with this workshop?" students answered that they learned to use Maple, to solve problems and to reason in mathematical and computer terms. They wrote that they have learned to schematize the process of solving a problem and some of the programming language; some students wrote that thanks to this workshop they have discovered a path they could be interested in for the future. Aspects that students particularly appreciated in the workshop were: group work, collaboration between students and teachers, problem solving, results comparison, the fact that it was not pure notionism, and that it was not a normal frontal lesson, the dynamism of meetings, the seriousness, clarity and availability of tutors and interactive components.

Conclusions

All the solutions created by students were very valid and full of interesting ideas. Students' answers to the final questionnaire are definitely positive and reflect what was said in the theoretical framework on the use of an ACE for solving a problem and in general for learning Mathematics through problem solving. The ACE allowed various solving strategies in different ways and the fact that they can be used simultaneously in a single environment has to help students in understanding and solving the problem. One of the things that struck students the most was the design and programming of interactive components that allow showing how results vary depending on the input parameters. This workshop was brief but can be an example of how it is possible to structure problem solving activities using an ACE. It is not important that students already know how to use an ACE before problem-solving activities, because they can learn to use it by solving problems, guided by the teacher, collaborating with their peers and autonomously, and exploring the necessary commands within the Help. Since the software offers multiple solution strategies for the same problem, the activity could continue with a presentation of the work done by the groups to their classmates and to the teacher. In this way, students would relate to others not only during the resolution of the problem, but they would also learn to present their ideas and support them in front of other people. Moreover, the problems used could apply not only to mathematical topics but also to other STEM disciplines. This experience shows how technology can be used naturally in ordinary teaching. It allows the teacher to rethink the teaching methods, allows the student to develop mathematical, digital and problem solving skills. The workshop, in terms of duration, topics and methods can be repeated in any type of upper secondary school class.

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