Je-LKS
PEER REVIEWED PAPERS
Vol. 14, n.3, 2018
ISSN: 1826-6223 | eISSN: 1971-8829
Journal of e-Learning and Knowledge Society
The Italian e-Learning Association Journal

FROM PRACTICAL TO THEORETICAL THINKING: THE IMPACT OF THE ROLE-PLAY ACTIVITY

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Keywords: Language, registers, written argumentations.

This paper concerns the analysis of the outcomes of a role-play activity aimed at fostering conceptual understanding of linear algebra for the first-year engineering students. The role of the teacher is considered in order to investigate about whether the designed role–play activity is able to affect the passage from practical to theoretical thinking. The analysis shows that, in spite of some difficulties related to the use of language, the students have improved their mastery of theoretical thinking.

for citations:
Pierri A. (2018), *From Practical to Theoretical Thinking: The Impact of the Role-Play Activity,* Journal of e-Learning and Knowledge Society, v.14, n.3, 7-15. ISSN: 1826-6223, e-ISSN:1971-8829 - *DOI: 10.20368/1971-8829/1375*

1 Introduction

The purpose of this paper is the analysis of the role-play activity; the outcomes of this analysis were already presented in our previous work (Albano & Pierri, 2014), in which the role was considered for investigating given competencies addressed by questions created by students.

Nevertheless, we now focus our attention on specific aspects of the students' reasoning in linear algebra. A number of students, indeed, tend to think by following practical rather than theoretical ways. "What is theoretical or practical is not thinking as such but *the relationship between thinking and its object"* (Sierpinska, 2005, p.120). From this point of view, this paper aims at analyzing undergraduate students' ways of thinking while playing the role of a teacher, who are required to pose some questions about specific topics of linear algebra.

In the framework of "Writing to Learn" (Morgan, 1998), as the role is played in an e-learning setting, it will be based on a written communication; hence the different forms of writing used by the students can be analyzed.

In the following sections, we are going to investigate the outcomes produced by the students and to discuss the subsequent findings with respect to the activity goal.

The questions addressed by the research are the following:

- How can the designed role–play activity affect the passage from practical to theoretical thinking?
- How can this kind of activity improve the students' writings going from abstract to algebraic-geometric language, the latter being characteristics of the language of mathematics? (Albano & Ferrari, 2013).

Finally, we try to draw some ideas for further works concerning the students' thinking.

2 Theoretical background

Linear algebra is an important field of study in mathematics, in particular for two principal reasons: on the one hand, it has a wide range of applications in different fields such as analysis, probability, etc.; on the other hand, it gives the students the opportunity for learning how to make mathematical abstractions. In particular, it provides them with instruments for graphical, numerical and symbolic representations that can be used in parallel or be chosen by the students according to their needs and/or personal preferences.

Notwithstanding its importance, teachers and students are aware of the fact that teaching and learning linear algebra is a difficult experience (Hillel, 2000).

Many are the researchers who conducted studies on linear algebra (Carlson, 1993; Nardi, 1997, Dorier, 2000, Pavlopoulou, 1993), and provided various theories in order to explain the reasons underlying the student's difficulties in understanding linear algebra issues and to present some suggestions about possible teaching methods.

Precisely referring to this persistent problem when teaching and learning linear algebra, Sierpinska (2000, p. 211) stated that

> "*We understood that for all the innovations that we made in presenting the theory to the students, we still wanted them to understand the same theory: the structural theory of linear algebra.... But the students in our experiments could not understand the theory because they appeared to want to grasp it with a 'practical' rather than a 'theoretical' mind*".

Sierpinska (*Ibidem*) assumed that in spite of a number of implementations for improving the teaching, some students continued having difficulties in understanding concepts in linear algebra. She claimed that this result depended on the inconsistency between the nature of linear algebra issues and the student's modes of thinking. From this point of view, this study aims at exploring undergraduate students' ways of thinking while solving specific tasks related to linear algebra topics, through a role-play activity.

Sierpinska (*Ibidem*) aims at identifying the characteristic of the students' way of thinking in linear algebra. Sierpinska reports three types of thinking modes: Synthetic-Geometric, Analytic-Arithmetic and Analytic-Structural, which derived from responses given by her students in linear algebra courses.

There are some fundamental differences among them, as the first one makes use of geometric representations, allowing the objects to be given readily (described through representations), but not to be defined, the two analytic modes instead, use numerical and algebraic representations. In these modes, the objects are defined through their definitions and properties.

This aspect is seen as strongly related to the diverse ways of representing objects in the linear algebra context. In parallel with the three modes of thinking identified by Sierpinska, Hillel (2000) three basic languages being used in linear algebra. A "geometric language" that refers to two and three spaces (points, lines, planes, directed line segments and geometric transformations), an "arithmetic language" that refers to n-tuples, matrices, rank, solutions of systems of equations and so on, "algebraic language" that refers to the general theory (vector spaces, subspaces, dimension, operators, kernels).

Basically, the theoretical framework of this research is twofold: Sierpinska's (*op. cit.*) modes of thinking and Hillel's (*op. cit.*) modes of describing and representing problems in the linear algebra context.

Several studies in Mathematics Education have shown that the difficulty for many students in studying mathematics can be due to their little ability to juggle between the everyday life meaning of a word and the formal use of the same word in mathematics (see for example Bardelle, 2010).

For such purpose, we planned to explore undergraduate students' ways of thinking during the role-play activity (Albano & Pierri, 2014), and to continue the work started in our initial research with the aim of showing how this kind of activity would engage the students with more "advanced" thinking (Morgan, 1998).

3 Experiment Setting and Methodology

If it is true, as Borba (2009, p. 463) claims, that schools *"seem to be resistant to technology"*, it is also true that "the online education has become a common feature" (Rosa & Lerman 2011, p. 69) at the university level. This is probably because higher education institutions aim to provide access to students with specific (for example distance, work or childcare related) needs.

Therefore, the use of technology in education may help reduce the gap between students' out-of-school and educational experience. The e-learning modalities may offer the possibility to personalize individual paths as well as activities involving a form of collaboration among students but also between students and teachers. For the purpose of our experiment, a group of voluntary freshman engineering students, involved in two intensive trimester-based modules in mathematics, was taken as a study sample. Indeed, our research was focused on the second module, which concerns topics covering linear algebra and calculus. The Linear Algebra course was composed of 60 hours, (5 per week), split into 3 hours of theoretical lecture and 2 hours of practice session. The Exam session consisted of two parts: the first one that included a written test having a compulsory nature for accessing the second part, and an oral exam (discussion); and the second part, where the student was required to master definitions and theorems (including understanding of the proofs). The final mark was the result of both written and oral tests.

In the context of the Linear Algebra course, we created and followed up a blended course delivered by a learning platform. The course was piloted with the voluntary group, composed of almost 70 first year engineering students.

The students, enrolled in the blended course from the beginning, could familiarize with the learning platform and the activities available on it, as part of their first attended lesson. They were also acquainted with the use of these activities to practice mathematics, though stating that they were not compulsory. They finally had access to a common area for them to exchange ideas, comments and questions about the activities in the platform. As regards the functionalities offered by the e-learning platform, we exploited a specific feature dealing with open-ended questions (tasks – “compito a casa”), suitably temporized. The temporization of this activity usually induces the students to carry out the assigned task in a fixed set time though, at the same time, gives them the possibility to choose when and where to complete the task. Precisely through this functionality, we could create the role-play activity, starting with assigning three different topics to each student taking part in the game, by playing a specific role. In other words, before starting with the experimentation we prepared a list of topics covering the Linear Algebra programme, for example Cramer’s Theorem, Rouchè-Capelli Theorem.

The students had to run a specified role (teacher, student, and teacher) for three days so that each cycle lasted nine days, three per role. For the first role, the student acted as a teacher requested to evaluate the level of knowledge of a topic through the formulation of questions referred to the same topic. In the second role, the student had to answer a question formulated by his/her colleague. Finally, for the third role, the student, still acting as a teacher, was asked to check the correctness of the work delivered in the previous phases.

After having distributed specific topics to each student playing the first role, the teacher assigned a web task, as in the following example:

> *“Formulate at least four questions that you think are useful to verify that a student has understood the statement and proof of Theorem X, as if you were a teacher who wants to evaluate the learning of the specified topic”.*

4 Analysis of Written Argumentation

In the following section, the work performed by the students regarding the first role is analyzed in the light of Sierpinska’s modes of thinking.

Let us consider a few protocols.

In the first iteration cycle, some students formulated the following questions, by using an Abstract language

> Q1: “Taking into account the various algebraic structures (G; σ), that we know, which of them can be defined as groups?”
> Q2:” “*Extension*” of groups in rings and fields”
> Q3: “Given a matrix A (m, n) and A’ its minor of order p, how many ways do exist for fringing with the lines of A?”
> Q4: “What elements define a row echelon matrix”?
> Q5: “Which of the two Laplace’s theorems states that the sum of the products of the elements in a row or column is always null”?

As we can observe, the formulation of the questions remains at a superficial and generic level. In fact, although it has required a verification of the statement and a proof of the assigned Theorem, what really appears is the inability of the student to enter into the details of the proof.

The above questions are expressed using specific descriptions of different objects but give no definitions; for example, this is true for "*what's the meaning of "algebraic structures?"* in question Q1, and "*Extension*" in question Q2.

In other cases, the questions are formulated by exclusively using symbols, that is without giving a definition of terms, as seen in "A(m,n)" in Q3.

Indeed, this even caused some difficulties when it came to formulate four simple questions, as highlighted by the students themselves.

In other situations, we can also find some student's attempts to copy the requested definition from their textbook. For example, in question Q5, the student affirms: *"the sum of the products of the elements in a row or column is always null",* though leaving out some details related to the matrix.

However, another aspect that specifically emerged from this activity is that during the individual tutoring sessions, it gave rise to a form of 'competition' among the students, who felt encouraged to do always better.

The following protocols show examples of their use of the Algebraic language

> Q6: "Solve the following linear system with the help of Cramer's rule"
> Q7: "Reduce the following matrix in row-echelon form S and calculate the rank"
> Q8: "Fixed a row-echelon matrix, what does the number of non-null rows correspond to?

We can observe an algebraic (operational) rather than abstract approach.

This approach can be regarded as a formal system designed to fulfill specific purposes, among which the opportunity of performing computations/procedures correctly and effectively.

However, also in this case, any form of theoretical thinking is absent.

During the last iteration cycle, we analyzed the protocols written by the students who had already formulated the questions from Q1 to Q5. A hint of a more complex language, probably associated to an analytic-structural way of thinking, is observed. See the following protocols:

> Q9: "According to the statement in the Gram-Schmidt's Theorem, what is the imposed condition on the dimension of the Euclidean space? And why?"
> Q10: "How can we use Steinitz' Lemma in the proof of the Base Theorem".

Q11: "What is the relationship between scalar product and vector norm?"

From the feedback received by both the tutor and the students, we can see how - despite some difficulties with the use of the language and the formalism for describing specific concepts still persist - it is possible to highlight a greater acquisition of theoretical thinking by the students.

This analysis allows us to draw some conclusions that are supported by:

- Feedback from students, after the numerous interactions between them and the tutor, both in a synchronous and asynchronous way.
- Progress made by clever students at the exams, observable at the end of each exam session.

As a result, the role-play activity has allowed the students to understand their educational goals compared with the operational ones, also improving their conceptual approach.

A possible reason for this achievement, could be found in the fact that this kind of activity has given the students a guidance for an in-depth analysis of the course topics; in particular, they tried to go into the details of the theorems' proof by overcoming the superficiality of the initial approach.

Indeed, as also claimed by some students during individual tutoring sessions, they adopted a critical attitude when posing a question, due, on the one hand, to the unicity of the method, and, on the other hand, to the its requirement to formulate at least four nonrepeating questions, helping them to study all the aspects of a given theorem.

It is also worth mentioning that some students used the teacher role for clarifying topics or particular steps of the proof for a specific theorem.

In brief, from the analysis of the protocols, it seems that the design of the role-play activity can help the students develop concepts and a more abstract ways of thinking. Indeed, looking at the questions made by the same students during the different cycles, we can observe a passage from the functional thinking to the set-theoretical thinking, where the utilization of the many functions passes from processes to objects.

Our analysis suggests the existence of theoretical thinking signals that find their expression through texts. Consequently, semiotic representation systems become themselves an object of reflection and analysis. Actually, as clearly pointed out by Sierpinska, acting at a meta-theoretical level constitutes the very essence of a theoretical perspective:

> […] Theoretical thinking is not about techniques or procedure for well-defined actions, […] theoretical thinking is reflective in that it does not take such techniques for granted but considers them always open to questioning and change.

> […] Theoretical thinking asks not only, is this statement true? But also what is the validity of our methods of verifying that it is true? Thus theoretical thinking always takes a distance towards its own results. […] theoretical thinking is thinking where thought and its object belong to distinct planes of action. (Sierpinska, 2005, pp. 121-23)

In the school context, the complexity of this meta-theoretical level seems to be ignored. At a university level, the professor has the duty to put into practice all the activities useful to develop possible ways of advanced thinking.

Conclusions and Future Work

In this paper, we have reported a work of outcomes analysis based on a role-play activity experimented by a number of first year engineering students, and aimed at fostering their conceptual understanding of mathematics. The activity has shown that introducing the students to theoretical thinking about mathematics is not such an easy task, in that it requires a long-term effort and has to take into account a great variety of contexts in which mathematics is studied and/or applied.

The transition from colloquial to ‘abstract’ mathematical registers is not a simple one but requires major changes in the organization of texts.

Therefore, we plan to continue the analysis of the other roles, in order to stress and ameliorate the students’ thinking way.

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