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E-TEACHING IN MATHEMATICS EDUCATION: THE TEACHER'S ROLE IN ONLINE DISCUSSION

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This article describes the cross-referenced results of two educational experiments on the use of social platforms in the teaching and learning of mathematics. These were conducted over the course of two years in two mathematics courses at secondary schools in the Campania region of Southern Italy and in two courses of Calculus at University of Salerno, in the south of Italy. It aimed to highlight the role that the teacher assumes in online discussion, revisiting their role as a transmitter of knowledge and taking on the role of facilitator in the acquisition of skills. The qualitative results seem to point to significant benefits in the processes of evolution in the use and coordination of semiotic representation systems. Thanks partly to the help offered by the teacher, the peer comparison between enhanced the transition both from common to evolved language and from interpsychic to intrapsychic functions, i.e. the transition from forms of social activity to forms of entirely individual activity.

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1 Introduction

All teachers build up their own teaching style, which may be based on many different factors, such as personality, principles, and cultural background. It may change over time but, in the absence of further stimuli, some features remain basically the same. The performance of a classroom activity involves the alternation of individual tasks and mathematical discussions. A mathematical discussion can be defined metaphorically as a polyphony of voices articulated on a mathematical object which constitutes one of the reasons for the teaching-learning activity (Bartolini Bussi & Boni, 1995). The educational scenarios of the 21st century are strongly influenced by factors originating outside the school which a teacher cannot ignore or neglect. New technologies enrich our knowledge and culture if they are properly employed and, if teaching is to become effective, it cannot disregard these tools with which our digital native students are so familiar. The novelty of the web lies in the interaction that takes place in real time between teachers and learners or, if the exchange is asynchronous, in the possibility to take advantage of the contents of the lessons at the time deemed most convenient by the student. Other advantages may be found; for example in multimedia, i.e. in the availability of audio, video or image files accompanying the text; in the opportunity to communicate with the teacher through a messaging system or to communicate with them via a webcam; in the possibility to check the progress made by each user (by monitoring the consultation of contents, the results of tests and the successful completion of each stage of the learning process). The research starts with an analysis of the evolution of e-learning up to the third-generation of distance education tools before describing the cross-referenced results of two educational experiments on the use of social platforms for teaching/learning mathematics over the course of two years in two mathematics courses at Secondary schools in the Campania region of Southern Italy, and in two courses of Calculus at University of Salerno, in the south of Italy. Particular attention is focused on the issue of conceptualization, which is justified whenever an item is mathematised, which may occur outside mathematics (mathematization of the real world) or within mathematics (function theory). Several experiments have led to the identification of a broad outline for conceptualisation discussions, which can be divided into separate phases: opening; expression of personal senses; constitution of meanings; cognitive dialectics; institutionalisation (Bartolini Bussi, 1995). The experimentation activity for secondary school students was organized online using a closed Facebook group of which a university lecturer, a researcher and students were members. This group helped to further students' understanding of some mathematical concepts by discussing them together. This activity did not have a fixed duration and each member intervened spontaneously on previous comments, as is usual in online groups. This made it possible to avoid authoritarian behaviour because the students were afforded considerable autonomy by the fact that the teacher acted as a participant and an observer of the quality of the discussion; it also allowed interventions to be carried out on the methodological level, enhancing the negotiation, sharing and stabilization of meanings. It also promoted a peer-to-peer relationship by fostering comparisons with the observations of classmates. The experimentation * involved about 200 students who used the Edmodo social platform during the Mathematics 1 and Mathematics 2 courses, integrated with *social network*, the *blogging,* and the *microblogging* (like Twitter) and the *podcasting* (Branchetti *et al.,* 2018).

These experiments, carried out using new communication strategies, made it possible to reformulate previously proposed concepts and produced clearer and more complete information for the reorganization of classroom activities. They also highlighted the difficulties facing the e-teacher who "is aware of the constant impact of ICT on education, school and society and is able to construct a personal vision/philosophy of learning and pedagogy suited to a knowledge society. This implies the achievement of a personal and intellectual maturity on the part of the teacher, which may take many years". The use of new technologies for teaching is thus an arduous task, but one that is necessary if we are to move properly in a constantly changing educational scenario. The results indicate that the use of social networks among students is widespread. In addition, students show a favourable attitude to those teachers who use social networks as a teaching resource. The frequency of use of social platforms turned out to be quite high among university students, while secondary school students seem to perceive social networks more as a pastime and participated in discussions only if so requested by the teacher. On the contrary, academic activities are frequently initiated by the students themselves. In both cases, the teacher's classroom activity seems to draw considerable advantages, especially in the teacher's perception of the difficulties encountered on some topics.

2 Theoretical framework

The term mathematical discussion was formally introduced into didactic research by Pirie & Schwarzenberger as "discourse focused on a mathematical topic in which there are original contributions from the students and interaction" (Pirie & Schwarzenberger, 1988). Examples of mathematical discussion can be found in the literature on the teaching of mathematics dating from the mid-1980s on. The kind of discourse in the discussion is predominantly of the following pattern: teacher's introduction – students' response – teacher's feedback.

The position of Richards (Bartolini Bussi *et.al,*,1995) also leaves the teacher's role unclear and, in the French scientific tradition, the teacher is assigned the role of moderator because attention is mainly focused on the role of socio-cognitive conflicts (Doise & Mugny 1981), i.e. conflicts that are generated in peer interaction when a strategy is explicitly contradicted by another person taking part in the discussion. Although these examples are different, they are united by the conception of the discussion as "a tool for building domains of consensus through class negotiation in which negotiation on a mathematical subject can take place. Can scientific concepts really be negotiated? Can students in a rich and stimulating situation reconstruct by themselves the quantity and variety of mathematical instruments developed by humanity over the centuries?" (*Ibidem*). The problem is twofold: on the one hand there is the consistency of the student's product with pre-existing knowledge and, on the other, the efficiency of the construction process. In this process the presence of an expert guide seems necessary but, in didactic research, the emphasis on the student's responsibility in learning is not counterbalanced by an emphasis on the teacher's responsibility in teaching (*Ibidem*). The approach to discussion, which aims to ensure that the expert teacher can provide a set of analysis and planning tools without reducing the students' responsibilities has gradually resulted in a distancing from the concepts of constructivist mathematical discussion like those mentioned above, where the teacher's role is unclear and in a search for inspiration from Vygotsky who, when speaking of internalisation, refers to interactions between subjects (teachers and students) who interpret different roles that must both be conserved and enhanced in the teaching-learning activity.

We can distinguish three different types of mathematical discussion:

The discussion of a problem seen as part of the overall problem-solving activity, in turn, presents two aspects: *solution discussion*, meaning the process of the whole class solving a problem expressed in words with the support of objects or images or visualized with application software; *balance discussion*, "meaning the process of information, analysis and evaluation of individual solutions proposed for a problem expressed in words with the possible support of objects or images or in the course of a discussion overseen by the teacher".

Conceptualization discussion is understood as the process of construction through the language of the links between past experiences and particular mathematical terms. This can be introduced by direct or indirect questions such as: "What is a number? What is a graph? Or why have many of you described this problem as a geometrical problem?" The conceptualisation discussion entails the interaction of the whole class overseen by the teacher and focussing on a word or a phrase in order to facilitate the expression of the personal sense given by the individual students to their experiences, to their products, to their processes (referred to by the word or the phrase in question) in meaning, just as it has been crystallized in its sensitive carrier (a word or an association of words) through the social experience of humanity. Several experiments have made it possible to define a broad outline for conceptualization discussions, divided into several phases:

Opening: in this phase the teacher introduces the discussion with questions such as "what do you mean? What does it mean?"

Expression of personal senses: this usually leads to the production of verbal texts perhaps accompanied by drawings or gestures that generally refer to previous experiences, all of which, the subject claims, are attributable to the word or phrase in question.

Constitution of meanings: this is guided by the teacher using the following technique: when a student clearly provides a relevant example that in itself contains the relationships between the fundamental features underlying the mathematical concept, the teacher generalizes the statement replacing the concrete determinations with more general expressions (object, thing instead of a concrete object...)

Cognitive dialectics between personal senses: this is achieved by the teacher explicitly inviting the students to produce statements that do not refer to particular cases (from sense to meaning) and to produce other examples of the same meaning (from meaning to a potential personal sense).

Institutionalisation: this is carried out by the teacher both by establishing (assessing) that certain links are pertinent and by explaining the meanings through their linguistic formulations, which are then recorded in writing on each student's notebook.

Meta-discussion: this is understood as "all those discussions that initially ask a question directly connected to metacognitive activity. It can be implemented by reconstructing the history of the class, often started by reading a passage taken from a previous discussion; discussions on the relationship between reality and mathematics; discussions on the method (what the sense of discussion is)." (Bartolini Bussi *et al.*, 1995).

Although these are different activities, in our experimentation they have been conceived as being ideally connected in a collective path.

3 Social learning and the Edmodo platform

In agreement with the fact that "every function of cultural development appears first on the social and then on the psychological level, firstly among people as an interpsychological category, then within the student as intrapsychological category" (Vygotskij, 1987, p.11), we tried to enhance peer comparison. For this purpose, we used the "Edmodo" social platform and a Facebook group of which all those enrolled in the course were members. Edmodo is a digital platform designed to work with groups of students in a protected environment.

Visually it has a central space where messages appear, enclosed between two service panels on the right and on the left, as in fig.1.

Fig. 1 – Homepage of Edmodo group of Calculus 1

Within the group, communication can be one-to-many (teacher to all, or a student to all) or discrete, (between teacher and student). In addition to dialoguing with the teacher, students can send attachments in the form of documents on which the teacher adds notes online and sends back to the sender (perhaps in one-to-one mode). Other useful tools that the platform offers are a shared library where documents, and images can be stored as well as a calendar reporting the deadlines for homework and test dates and the possibility to create quizzes and manage assessments while protecting privacy. The e-learning approach allowed the students to continuously interact with the teacher and the tutors and made it possible to activate and promote peer-education processes through a constant interaction between all members of the class group.

Students were provided with educational material through the Edmodo platform and simple exercises were proposed for class discussion. The learning material provided to the students and the related exercises aimed to encourage the students to engage in a discussion on the web in such a way that the teacher was able to understand the student's point of view on the topics to be discussed. The classroom lesson stemmed from the students' knowledge and was also oriented on the basis of the most common mistakes or any misconceptions documented and analysed in previous research arising during the debate.

We also tried to promote Self-Regulated Learning (SRL) defined as a "set of processes through which students personally activate and support cognition, emotion and behaviour systematically oriented towards their own objectives" (Zimmermann & Schunk, 2011, p.1). Interaction in a virtual community also proved useful because it facilitated a more natural transition from a colloquial linguistic register to a formal linguistic register.

As can be seen from Figure 2, even when dialogue between students used colloquial language it still allowed the teacher to identify difficulties in how students tackled the exercises.

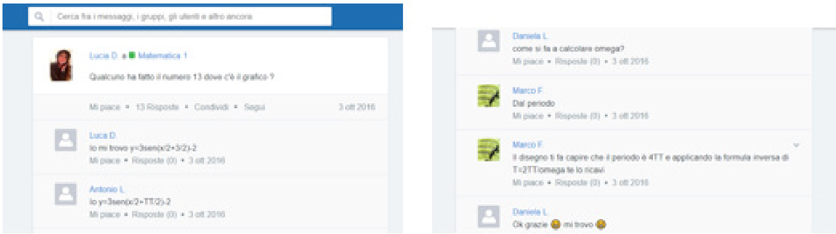

Fig. 2 – Some examples of student dialogue windows

The shared posts and comments left by the students made it possible to identify "Just in time" thematic groups to be studied in-depth and also to intervene on some misconceptions.

4 Experimentation using Facebook and Moodle

A first experiment used a closed Facebook group as a work environment, comprising students from 2 fifth-year (final year) classes of a *Liceo Scientifico* (secondary school focussing on scientific subjects). The topic for discussion was the concept of limit, which is fundamental in mathematical analysis but whose essence is not always understood by students, as emerged from the teaching/learning experiment. In a recent research article on the teaching of mathematics (Tall & Katz, 2014), one thing the authors often asked themselves was if the notion of dynamic change worked in Cauchy's lessons, why is the concept of limit today usually presented to students using definitions that are far beyond their experience? Moreover, the authors wondered if mathematical thinking can develop from the perception of a concept up to more sophisticated forms of reasoning. In looking for an answer, they referred to different theoretical perspectives and theories on the development of mathematical thought (Sfard,1991) which have focused on the way in which human beings perform mathematical operations, such as addition, division, calculation of the limit, derivation and integration. They concluded that, in each phase, a process takes place over time and produces a result that can also be conceived of as a mental entity, independent of time. Cornu (1991) then described how students think of the concept of limit as a process of reduction, as the production of an object that is arbitrarily small but not zero, defined as the *generic limit*.

The reading of the work Katz & Tall made us reflect on number of questions:

why is the concept of limit introduced to students with definitions that are far beyond their experience? Why is the concept of limit difficult to learn and to teach? The answer I have come up with is that very often the concept of limit is introduced in a "sophisticated" way. There is no right or wrong approach to helping the student understand this concept, but rather the "formal" and "non-formal" approach to the theory of limits depends on the context in which the students finds themselves. For this reason I tried to understand how to integrate the two aspects (formal and non-formal) and, while reflecting on this, I thought of tackling it in an environment with which students were familiar – Facebook – as I felt that continuing to teach our students ideas on limits that are implicitly based on a formal, axiomatic approach will give life to a series of generations transmitting ideas that fascinate few but confuse many in a society which has to serve a wider group of students to ensure wider participation within that group.

The decision to use Facebook was dictated by its ability to fill gaps in a timely manner and thus avoid a sense of defeat during the study of Mathematics, and also facilitate interaction with teachers and peers, thereby reducing workload and anxiety. This is made possible by the simplicity of the interface, the possibility of viewing the group on mobile devices and the straightforward feedback system: Facebook users can also easily enter comments or observations at any point of the discussion through a simple link and express their appreciation by clicking on "Like".

Before starting the online activity, a face-to-face meeting was held in order to introduce the activity and to allow me to better understand the students' relationship with mathematics, with respect to non-cognitive variables that influence the learning of mathematics (Di Martino & Zan, 2001). To this end, they were asked to write an essay entitled "In your opinion, what is mathematics?" in one hour. After this meeting, all interactions between students and between students and the researcher/teacher took place exclusively online, within the closed group "Noi e la Matematica" (Mathematics and Us), a name, chosen by the students themselves during the first meeting.

A second experiment was carried out using Moodle. This choice was dictated by the consideration that, unlike Facebook, Moodle is a purpose-built teaching tool that supports active learning, i.e. it makes it possible to build a topic, communicate it, collaborate on it and share it. It also offers a variety of ways to manage interactions through different types of forums and this allowed me to redesign the discussion activity using the Questions and Answers Forum, where each student could see the answers of others only after posting their own, and to add a new topic and reply to comments on the already active discussions. It was thus possible to rule out fake participation, in which a student merely shares the thoughts of others.

This activity took place with fourth-year students, again from a *Liceo*

Scientifico. It began with a face-to-face meeting during which a discussion was started on the concept of function, a topic that they were currently tackling with their class teacher. Subsequently, the activity continued on the platform. The topics covered were those that were taking place in the classroom with their regular teacher.

A third experiment again used the Facebook social network and was conducted on university undergraduates, specifically first-year engineering students who were attending a mathematics course. The experiment was open to all students who wanted to review some topics considered as key prerequisites for subsequent courses. However, particular attention was focussed on the planning of activities aimed at supporting students with educational debits (OFA). The course included 25 hours in class (5 hours a day for 5 consecutive days) and was supported by activities on Facebook, which continued after the end of classroom lessons, until the two tests were held. (about two months). The 63 students taking part in the course were enrolled in a closed Facebook group called "Precorso IngInf". Discussions were activated by posting entry test questions from previous years. In the test, multiple-choice questions were used, while in the group the questions required the candidate to justify the chosen answer:

"Post your answer explaining what reasoning or procedure you followed to reach your answer"

After the students had answered, especially in the case of a wrong answer and an incorrect procedure, they were asked further questions specifically created to stimulate the student to find the mistake made and to identify the correct procedure. After completing the activity, a questionnaire was posted in order to assess the students' feedback on their activity. The questionnaire was administered after the tests in order to make sure, as far as possible, that the responses were not somehow influenced by presumed with the recovery of OFA credits.

Conclusions

The experiments were conducted on 74 secondary school students and 212 university students divided into three groups who interacted with each other and with teachers on closed Facebook groups, through a MOODLE platform and Edmodo.

The activity involving secondary school students on Facebook was quite successful as regards both the active participation of students and the recovery and consolidation of mathematical skills. The students perceived Facebook as a virtual place in which to meet also with a view to discussing the studied topics.

The discussion on the Moodle platform had a greater educational success than that on Facebook because Moodle is a tool designed specifically for teaching. Moreover, thanks to the use of the questions and answers forum, it was possible to avoid answers like “I agree with...”. The active participation of some students highlighted an evolution in their way of reasoning that led them from mere procedural execution to argumentation. There was also a blended use of the technologies when they started posting photos of exercises completed in exercise books or on the multimedia interactive whiteboard.

Discussion with university students on a closed Facebook group worked well. There was a very significant participation of those who did not need to recover an OFA debit but who took part because they considered this opportunity as something that would allow them to further investigate the issues in question. This produced a double positive effect: on the one hand, the comparison between peers led to improvements in the topics in question and, on the other, these students served as tutors for their weaker peers, thus providing an additional resource for the teacher, whose presence was widely appreciated for the interventions and the interactions within the group. The onset of discussion in the more advanced phases showed how students are in favour of participating in activities with an instrument that is familiar to them. Obviously, the evolution in the individual reasoning was not the same for everyone. The teacher’s interventions never aimed to give the exact answers but were mainly intended to support critical thought, which led some students to enhance their reasoning process, moving from the procedures to the reasons underlying them and to the identification, understanding and recovery of any errors made by them or their peers. The teacher’s interventions also resulted in an evolution in the use and coordination of several systems of semiotic representation: for example, some students began to complete exercises not only by using what they remembered but also by exploiting graphical representations. However, there were also interventions by students who, on the other hand, only posted work in which there were calculations. This seems to underline the beneficial effect of having a heterogeneous group, as it was at the beginning, in which there were students with different levels of learning, both people in difficulty and students who did not have any debits. The interaction between these learners seems to have been more fruitful, precisely as Vygotskji intended, because the stronger students represented a zone of proximal development for the weaker ones

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