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PERSONALIZED LEARNING IN MATHEMATICS

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This work shows an innovative solution in order to enable the predisposition to the mathematics, by using auto-regulation of learning objectives, personalization for obtaining, in such a way, a learning path more compliant to the learner's needs. In this paper, we focus on an e-learning module, aimed to foster theoretical thinking in facing linear algebra problems. It has been developed assuming an integrated approach that combines structural and operational view of a concept. It consists in interactive and dynamic learning activities, based on the duality process-object, suitably formalised in an appropriate representation of mathematical operational knowledge. The module is available into the e-learning platform IWT where new interactive Learning Activities show the potentiality of the symbolic calculus of the integrated Wolfram Mathematica environment. Due to the given knowledge representation and intelligent features of IWT, these learning activities can be delivered within a personalised learning work, according to the individual

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needs. Open-ended tasks have been used both to complete and to validate the module.

1 Introduction

This paper is framed in the field of mathematical e-learning, aimed to investigate what advantages e-learning can offer to mathematics learning.

Nowadays citizen are required to become somewhat *competent* in mathematics, which means able "to analyse, reason and communicate effectively as well as to ask, solve and interpret mathematical problems in a variety of situations" (OECD, 2004).

Many authors have tried in a hard way to define what *competence* in mathematics is, but it is clear that goes beyond the cognitive factors. They are heavily involved both metacognitive factors, such as awareness of their own cognitive resources and their thought processes and the ability to *self-regulation* (Zan, 2000; Zimmerman, 2000), and non-cognitive factors, such as attitude, acceptance of the stimuli to use their knowledge and desire to integrate them when necessary (D'Amore, 2000).

Competence in mathematics is not something to teach; rather it is a long-term goal of the learning process. However, it requires knowledge of the mathematical domain both declarative-propositional (*knowledge*) and procedural (*skill*). We emphasize that both are equally important. Given a problem, the knowledge allows to understand and analyse, then provides the necessary theoretical results, the reasons and background that allow to formulate and validate theoretical approaches useful to address the problem.

However, knowledge alone is not enough without procedural skills. The latter produce results and practical consequences to make concrete specific theoretical arguments. At the same time, skills without knowledge can lead to the application of correct procedures in erroneous contexts, coming to meaningless results.

The development of self-regulation in the students requires a new conceptualization of learning environment (Azevedo *et al.*, 2005) such as cognitive artefacts (Agostinelli S., 2007) that enable the student to the control of the essential dimensions of the learning (Kemp *et al.*, 2009), to the orientation and selection of the competence's objectives (Nussbaumer, 2008).

Some empirical evidences affirm: "*that learners typically do not adaptively modify their behaviour, thus suggesting that they engage in what is called dysregulated learning. Dysregulated learning is a new term that is used to describe a class of behaviours that learners use that lead to minimal learning. A lack of self-regulatory skills is the main obstacle to adequate regulation and therefore deficient learning gains and conceptual understanding*" (Jacobson & Azevedo, 2008).

In this context, the *MatematicaFacile.it* initiative has as *leitmotiv* to change the forms and methods of mathematics learning through a *pedagogical driven* solution that integrates, in a functional way, technologies and teaching models. That in accordance with the principles and learning conditions and guidelines that revolve around research on *metacognition* and *self-regulated learning* (Azevedo, 2005; Mayer, 1999).

The last goal is to improve cognitive processes and promote the vocations in math in high school students by helping them to overcome the obstacles that the academic path could show. Learning, especially that related to "conceptually-rich domains" (Azevedo, 2005; Lin, 2001) as indeed the mathematical domain, requires to be able to use environments that support activities of self-regulation of the objectives, personalization of the learning paths in order to obtain specific paths that meet the requirements of the individual (Vovides *et al.*, 2007; Sanz de Acedo Lizarraga *et al.*, 2003).

In this respect, e-learning can give advantages (Albano & Ferrari, 2008; Albano, 2011; Bardelle & Di Martino, 2012; Descamps *et al.*, 2006):

- it allows to offer flexible *just-for-me* courses;
- the teacher can following-up and coaching his/her students anywhere at any time;
- the learning resources are richer and there are some evidence of deeper understanding;
- it is possible to handle various semiotic representations;
- learning experience can be tailored according to individual student difficulties and needs.

The technological solution at the basis of MatematicaFacile.it is the learning platform IWT (Intelligent Web Teacher). It is an ontology-based system composed by agents operating on three main aspects: didactical knowledge, student model and planning procedures, that allow the generation of didactic paths that are personalized taking into account the cognitive state and the learning style of the student (Mangione & Pierri, 2009; Capuano *et al.*, 2009; Gaeta *et al.*, 2009).

In order to better work on the domain of mathematics, IWT has been properly integrated with a specific plug-in that have allowed the use of Wolfram Mathematica environment and the creation of new interactive learning objects that exploit the potential of symbolic computation.

In this paper, we present an e-learning module, which consists in a Mathematica application, concerning linear algebra courses for freshman in engineering faculties at University. It aims to foster theoretical thinking in facing linear algebra problems. The e-learning module has been developed assuming an integrated approach that combines structural and operational view of a concept (Albano, 2013). Such module takes advantages of the available CAS features of Mathematica and WebMathematica in order to generate interactive and dynamic learning activities, named WebMathematica learning activities (WM-LA). The module has been integrated into the e-learning platform IWT and, due to *intelligent* features of IWT, the WM-LAs can be delivered within a personalised learning work, according to the individual needs.

The module have been experimented within a wider proposal of e-learning support for freshman attending linear algebra courses in engineering faculty at the University of Salerno. Open-ended tasks have been used both to complete and to validate the module.

2 The technological framework

Intelligent Web Teacher (IWT) is an innovative solution for learning. Its framework follows a *pedagogy driven* approach: its focus is on the learner and on his/her curriculum planning; its technology offers many learning opportunities for different targets and contexts (Capuano *et al.*, 2009; Mangione & Pierri, 2009).

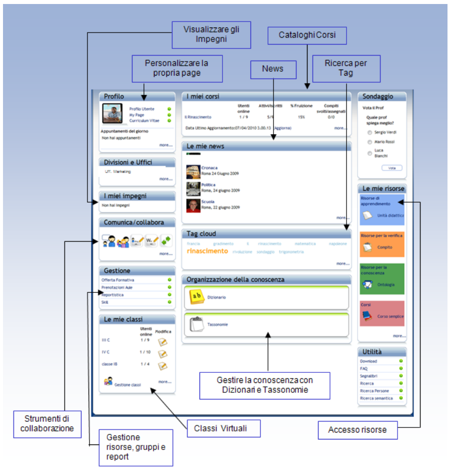

Fig. 1 - IWT Home Page

As a benefit, IWT offers custom learning experiences by taking into account the context, the learning domain and the principles of the individualization based on cognitive state and learning style of each user. Moreover, it allows also user to become *self-directed learner* (McCombs, 2001).

To create a personalized course IWT needs an ontology, a set of learning objects described by means of metadata and connected to the node of the ontology itself. The objective of a personalized course is a set of concept (*target* *concepts*) of the ontology. IWT applies its algorithms on the graph of the ontology and creates the sequence of concepts to treat (*learning path*) that are able to reach the fixed learning objective. Then, IWT personalizes the *learning path* by respecting the cognitive state of each user. In other words, IWT leaves on the course for each user only the concepts that are not in the cognitive state of this user. Finally, IWT finds the minimal set of learning objects able to explain the concepts left in the *learning path* by respecting the learning preferences of each user. This means that IWT delivers to each user a suitable personalized learning experience (in terms of sequence of concepts and learning objects) able to respect his/her previous knowledge and preferred learning style and to lead that user to the fixed objective (Gaeta *et al.*, 2009).

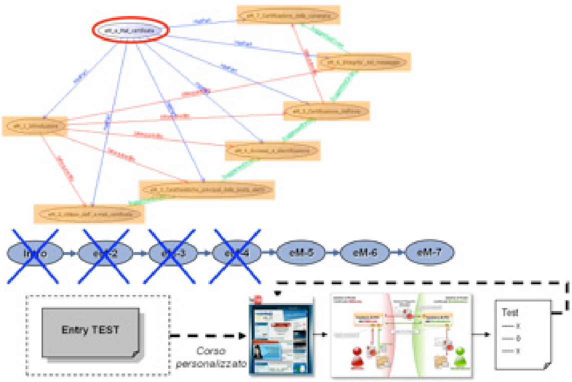

Fig. 2 - Construction of the Learning Path

In IWT, the *assessment* certifies the roadmap to a learning objective. The assessment results allow IWT understanding which concept may need some explanation more by using, if available, content having a learning style different from the used one (Miranda *et al.*, 2013; Capuano *et al.*, 2009; Mangione & Pierri, 2009).

IWT is able to support different pedagogical models: s*ubject-based learning, narrative learning, problem-based learning, collaborative learning, self-directed learning, inquiry based learning*, etc. It is possible to integrate both formal and informal activities by applying many educational strategies: *lesson, tutorial, modeling, simulations, storytelling*, etc. Users may follow different approaches: *competency-driven, objective drive*n, etc. Learning activities may follow different ways: *e-learning, blended learning, mobile learning*, etc. IWT delivers learning experiences by guarantying completeness, pedagogical expressiveness and personalization.

Potentialities of IWT allow mixing together technologies and models so to face all issues related to the teaching and learning Mathematics. The objective is employ IWT to improve the knowledge processes and foster the growing of mathematical vocations of the students by helping them in the high school to climb over the problems at the beginning of the scientific university travel

To this day, there are no solutions able to give a valid answer to this problem and to respect the *conditions of learning*. Often, they are *textbook* with interactive exercises and applying the *drill and practice* approach. Also valid software applications, like *MyMathLab*[1], cannot assembly richness of the multimedia content together with the intelligent and dynamic construction of learning path for a student, for his/her learning style, for his/her own objectives. In general, these solutions are oriented to a directive and deterministic vision of the learning experience without any employing of models mixing both *teacher-centered* and *student-centered* approaches.

To support the development of knowledge, the ability on self-assessment and on self-planning, the learning solutions should support users to be drivers of their changes by means of their own profiles, preferences and needs. It is necessary to employ the right models and methodologies in order to create an effective and personalized experiences for learners, as better as possible.

3 The mathematics education framework

According to Sfard (1991), learning mathematics consists in student's construction of *conceptions*. Building blocks of mathematics can be referred as *concepts* meaning its official and formal mathematical definition within *mathematical knowledge*, and as *conceptions* meaning the person's mental, internal, private representation and understanding (Sfard, 1991; Harel *et al.*, 2006). Sfard distinguishes two complementary view of a conception: a *structural* one, as abstract object, and an *operational* one, as processed, algorithms, actions. She also states the importance of being able to handle both the views, to understand deeply mathematics, whatever the definition of understanding we assume. She notes that the construction of structural conceptions goes through the acquisition of operational conceptions, in a hierarchical three-step path:

- *interiorization*, where the student becomes skilled in performing procedures, first on more and more low-level objects and then just mentally performing;
- *condensation*, where the learner is able to pack long sequences of operations into a block, no longer needed to fully detail;
- *reification*, where finally a new "conception" arises. Such path highlights the importance of considering the duality of process (operational) and

[1] http://dev-mml-it.pearson-intl.com/

object (structural) going back and forth flexibly between them.

To this aim, Gray & Tall (1994) coined the term *procept* as an amalgam of process and concept (object). They distinguish between process and procedure: the former is a step-by-step algorithm where the user has to complete each step before to go on, the latter refers to more procedures, having the same global effect, seen as a whole. For instance, "solving a linear system" is a process which corresponds to various procedures such as Gauss algorithm, Cramer rule, etc. The interiorization of procedures can brought to their birth as mental object, able to be manipulated and becoming *structural conceptions*. When procedures-processes-symbols amalgamate, we have a *procept*. We can refer to a procept as both a mental conception and processes to be performed with the referred procedures. Thus the procepts allow thinking about mathematical conceptions, not only to perform mathematics.

4 The WebMathematica Learning Activities

The above theoretical and technological frameworks has been assumed as guide to design an e-learning module. It provides the students with learning activities, named WebMathematica learning activities (WM-LA), consisting in linear algebra problems to be faced, aimed to foster the learner's construction of operational knowledge using a structural approach.

Besides the well-known students' difficulties in the passage secondary-tertiary, difficulties in linear algebra can be ascribed to three causes: the axiomatic approach in teaching, which is perceived by students as superfluous and meaningless; the use of many different systems of semiotic representation; the need for developing *theoretical thinking* as opposite to *practical thinking* (Maracci, 2008). The growing of the number of the students together with the reform of the University courses (affecting both the number of the face-to-face classes and the syllabus) do not allow the teacher to carry out appropriate didactical actions to overcome students' difficulties.

4.1 The representation model for operational knowledge

We have used the definition of procept in order to represent the operational knowledge in a cognitive structure. Analogously to Crowley & Tall (1999), we consider both a web metaphor and a hierarchical structure, where the web nodes (parent nodes) are the procepts, and the hierarchical nodes (child nodes) are the various procedures associated to each procept, connected by edges. The parent nodes arise from the *conceptual compression* (Crowley & Tall, 1999) of the child nodes intended as the process allowing the transition from conceiving separated ideas to thinking at one idea with various aspects. So the procepts constitute what Barnard & Tall named *cognitive unit* (of operational kind), that is a piece of knowledge which can be manipulated as a whole, having a wide cognitive power, thanks to the internal edges (connections with child nodes), seen as different aspects of the same entity. This idea of cognitive unit has some common features with the idea of schema (Dubinsky, 1991). A difference can be that in the latter case processes and objects correspond to different phases.

The two levels of the representation of the operational knowledge are the following. On one level, we consider a graph whose nodes are the procepts corresponding to operational cognitive units, and the edges referring the relations among the procepts.

For instance, considering the pre-requisite relation, the procept "dimension of a vector sub-space" requires the procept "rank of a matrix", because, whatever the representation of the space is, the computation of its dimension requires being able to calculate the rank of a suitable matrix). Each node is then connected at a lower level to child nodes corresponding to the procedures associated to the procept (e.g. different methods to solve a linear system).

Going into more details, there can be connections between a procept (parent node) and a procedure (child node) of another procept, as shown in figure 3. In fact, in the case of the procept "determinant", it does not matter how to compute it – that are its child nodes, this procept is connected as a pre-requisite to the child node representing the Cramer procedure (thin dashed line).

Considering only the procepts web, we have introduced some added edges: if the procept 1 is connected to a child node of the procept 2, then an edge connects the procept 1 to the procept 2 (bold dashed line). These new edges will be useful to exploit some features of the platform IWT used for the implementation of the e-learning module.

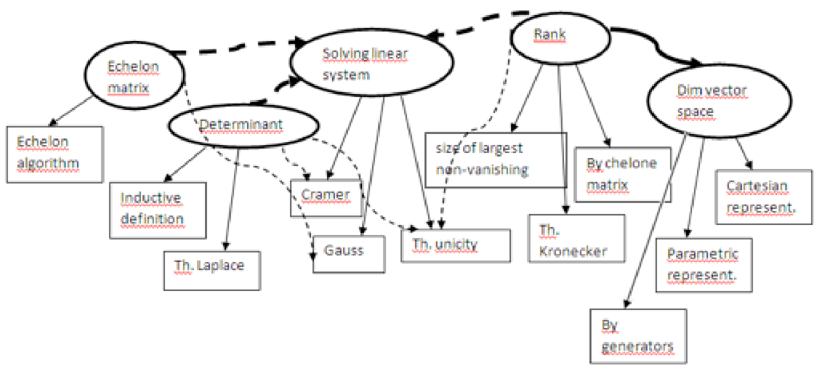

Fig. 3 - Operational knowledge representation

4.2 The rationale of the WM-LA's design

Exploiting the above knowledge structure, we have designed the WM-LAs. Each of them consists in more interactive and dynamic problems (PWM). Each PWM refers to specific procedures, that is step-by-step algorithm. The conceptual compression of more PWM gives rise to a WM-LA, thus the latter correspond to the parent nodes and the former to the child nodes. Thus, the WM-LA refers to a cognitive unit that is something to consider as a whole. This is why the compression can be generated according different criteria, such as:

1. compression of more PWM corresponding to one process and more procedures interchangeable (e.g. Gauss algorithm, Cramer rule) or mutually exclusive (e.g. because dependent on the representation of the problem data, as in the case of the computation of the basis of a vector sub-space where the algorithm differs according to the given space representation);
2. compression of more PWM referred to procepts which can be collected in one cognitive unit (e.g. matrix operations refer to various operations).

Each PWM defines a certain type of problem, but it is not a unique problem, thanks to the CAS power of Mathematica. In fact, for each type of problem, we have defined the structure of the problem (*template*) using Mathematica programming features for implementing a procedure handling symbolic data. When the student asks for a PWM, an instance of the template is generated *on the fly*, so that numerical values take the place of the symbolic data. Particular attention has been paid to the generation of the problem's data in order to make the problem effective from the education viewpoint. The only use of random function in most of the cases generates problem with hard calculations.

The template contains programming instructions to guide the student to solve the given problem step-by-step. What is important is that the design of the step-by-step algorithm refers to the definition of procept and so to its cognitive power to be able to think about operational conceptions, fostering actually the students to starting off structural thinking. In fact, we have defined "step" in a PWM either an atomic action (not classifiable as procedure or process) or a process (corresponding to an existing WM-LA, dashed line in Figure 3). For instance, often solving a linear system is a step in a PWM. This way the student is required to think about the process and then he/she is free to perform the step according to his/her preferred procedure.

The template also contains instructions for assessment. At each step of a PWM, the student is required to interact with the module, giving an answer, and the software gives automatic assessment. The assessment is performed through suitable Mathematica routines. For each PWM a deep analysis of the possible errors has been done, based on the availability of plenty of protocols (written exams), in order to design assessment routine able to detect various kinds of errors (calculations, logical inconsistencies, theoretical gaps, etc.). A list of errors and corresponding feedback has been drawn up. The symbolic power of Mathematica has been exploited to check equivalent answers and also in this case suitable check routines have been designed (e.g. two different bases of a vector space). According to each error, a personalized hint is given to the student.

5 The interactive assessment

The interactive exercises described in the previous section have been modified so that to be automatically assessed by the platform and then to become complex interactive tests which allow to assess in a suitable manner the skills acquired by the student. The number of the steps and the level of difficulties is various, then the assessment of each exercise, which consists in a mark between 1 and 10, cannot be always assigned in the same manner for all of them. It needs criteria that can take into account the structure (that is the number of the steps) more or less complex of the current exercise and at the same time the intrinsic difficulty of the exercise too. To this aim, the teacher for each type of exercise has defined an assessment algorithm. It allows IWT to trace both how many times each step has been tried before individuating the correct answer and if during the single step the solution has been visualised to go to the next step.

The assessment has made according to the following criteria:

- Each step of an exercise has an initial score equal to 10 points, to be confirmed, if the student answers correctly, or dropped off, if the answer is incorrect. The score is set to 0 if the student read the correct solution without any attempt of answering;
- The teacher can establish the number of meaningful tries for answering. Besides such number the student can continue moreover to give possible answers but any score will be given;
- Each failed try will drop off the initial score of the step (that is 10) of a suitable quantity chosen by the teacher, until 0 which corresponds to the last step considered meaningful by the teacher;
- Each step will have a fixed weight in the context of the whole exercise. The teacher will assign at each step a certain importance, which will be quantified by means of a decimal number between 0 and 1. It is fundamental that the sum of the various weights assigned at each step will have 1 as sum;
- The score of the exercise is equal to the sum of the scores of each step.

Definitively, according to the considered algorithm, it is fundamental fix

the following parameters:
- Number of steps for each exercise;
- Number of tries for each step;
- Method of dropping off the score in case of error for each step;
- Weight for each single step.

The interactive exercises can be seen as a complex entity through which we can evaluate pleasure, learning and change (the first tree Kirkpatrick'levels – Kirkpatrick, 1996)[2].

In order to put in practice these levels, it needs to test the following aspects:
1. the students-teachers appreciation of the instructional activity;
2. the knowledge and skills acquisition by the student and his/her level of exhaustivity after the learning process;
3. the application of the knowledge acquired through the experience in resolving real-life problems.

6 The Interface

The interface of webMath-LO requires the interaction with the symbolic mathematical notation. The user must be able not only to display mathematical formulas but also to interact directly with the LO through the inclusion of data that will then be evaluated by Mathematica. This need led to the creation of models for putting data (*template*) and indicators for the spaces to be filled for the writing and editing of formulas (*place holder*).

In fact, in many cases, the introduction of *template* and *place holder* allows to help the user in the writing of symbolic mathematics. Some examples of mathematical entities that have well-defined structures are: matrices, integrals, derivatives, roots, exponentiation, etc. When the user must use one of these components, the *template* will facilitate the data insertion, while the *place holder* will indicate to the learners the boxes to fill for complete formulas and mathematical expressions.

An example of a template for the matrices is shown in the Figure 4. This template includes the typical form of the matrix enclosed in parentheses with the place holder for the insertion of the matrix elements arranged in rows and columns.

[2] According to this approach four levels of evaluation are proposed. These are (1) Reaction, (2) Learning, (3) Behaviour, and (4) Results. While the levels one and two mainly refer to formative evaluations, the levels three and four are summative evaluations. In particular, the four level is used to determine the effectiveness of learning, in order to improve future programs and to eliminate ineffective programs.

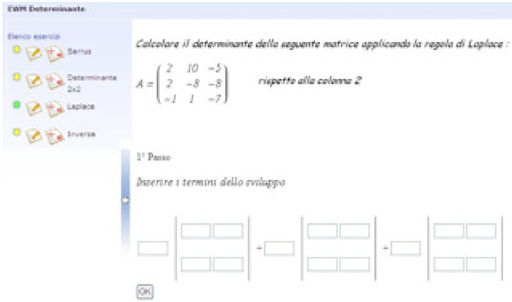

Fig. 4 - Example of exercise

7 The implementation in IWT

The WM-LA has been implemented in the e-learning platform IWT. The implementation of the WM-Las has been made possible by means of suitable Plug-In Driver and Object Driver, which give an interface to interact on one hand with the users and on the other hand with Mathematica. Such a way, the WM-LA can be managed as any learning resource in the platform. What we want to focus in this section is the exploitation of the previous given representation of operational knowledge. Using IWT ontologies, we have implemented the above representation of the operational knowledge. According to the theoretical model, the nodes of the ontology have been identified by the procepts referring to the WM-LA. This means that the ontology implements only the higher level of the cognitive structure in fig. 1. The lower level has been implemented as a learning resource PWM. Thus, the edges between the nodes correspond to the bold lines. More precisely two node are linked by an edge if a relation stands between the two procept (continuous line) o if a relation stands between a procept and a procedure related to the other procept (dashed line). At each node of the ontology has been associated a WM-LA, implemented as aggregation of more PWM, each of them associated to a child node in the cognitive structure previously designed. The association between nodes and WM-LA has been stored in the metadata of the WM-LA.

Thanks to an *intelligent* features of IWT (Capuano *et al.*, 2009), it is possible to exploit the above implemented knowledge representation in order to delivery unit of learning tailored according the learner needs and cognitive state.

8 The technological architecture

We may integrate IWT with numerical and symbolical computation engine by means of its *Plug-in Drivers* and *Object Drivers*.

The *Plug-in Driver* is the interface between IWT and an external engine: the Plug-in indeed. The *Object Driver* is the learning resource able to deliver functionalities and services of the external engine inside a learning path.

In this case, the *Wolfram Mathematica* platform by mean of its *WebMathematica*, became a *Plug-in* of IWT. We developed a specific *Plug-in Driver* and different *Object Drivers* able to use some services of Mathematica and create learning resources to include in the IWT courses. In particular, we created the resource type *webMathematica Learning Object* (webM-LO) able to allow users to access to functionalities of the *Mathematica* engine directly from the course they are following inside IWT. The teacher has to create *Mathematica* packages and all the visual interfaces to invoke them; then, he/she has to upload everything in the repository of IWT and fill-in the related metadata fields able to describe them from the semantic and technical point of view. When everything is done, IWT, and in particular its intelligent module LIA, is able to use also this kind of resources in the personalized courses when the features of the users match with the feature of these resources.

In fact, during the delivery, the learner may receive a Mathematical object inside a learning path. When it happens, the learner may access to this one and use it as a normal object created by *WebMathematica*. Meanwhile, IWT tracks the interaction between the user and this kind of resources, collects results and updates the cognitive state and the preferences of the user.

9 Validation of the model

The dual nature of the mathematical conceptions (*structural* as abstract object and *operational* as algorithms and actions) inspired the model designed in the webM-LA. As stated before, the construction of structural conceptions goes through the acquisition of operational conceptions, in a hierarchical three-step path (*interiorization*, where the learner gains skills in performing procedures, *condensation*, where the learner gains ability in creating block of operations, *reification*, where the learner gets a new “conception”).

The validation of the proposed model refers exactly to the term *procept* as an amalgam of mental conceptions and processes to perform. In the webM-L, each step is a procept: in fact, in this case it is considered as a whole, that is as a mental conception.

The experimentation started by supporting traditional linear algebra classes by means of personalised distance courses, automatically generated by IWT and further personalised by the students themselves, and individual teacher-driven learning activities (Albano, 2011). The tailored linear algebra course delivers to the student the most appropriate learning resources, according to his/her needs and preferences and tries to get them to the *procepts* of this mathematical subject. Anyway, the learner can explore alternative learning resources presented in IWT: hypermedia, static and dynamic problems, videos, animated slides, quiz are available for linear algebra.

Activities such as those described in this paper produce long-term outcomes that cannot be fully seen following the students just for a school-term. Anyway, in order to validate the e-learning module and to get some feedback of its efficacy with respect the initial aims, some open-ended tasks have been proposed to the students attended the e-learning course. Each task proposes a linear algebra problem. The request is not to write down the detailed solution of the problem, but the student is required to solve the problem by pen and paper and to give as answer to the task the description of the solving strategy used and the results of the main steps of such strategy. The request has been formulated in such a way that the student act as a teacher in order to make clear "to a friend how to solve this problem". Once the student delivers his/her answer, the teacher-tutor acts in the role of the friend, beginning a discussion aimed to promote reflection on the work done.

From a quantitative point of view, we note that 70% of the students attending the linear algebra course has taken part in this kind of task, with a little decrease as the discussion goes further.

From a qualitative point of view, we have noted what follows. At beginning, most of the students were successful in make clear the structural thinking underpinning the solution, but they also report more computations than needed. Some students in explaining the process refer to objects of their paper as they are speaking face-to-face synchronously with their friend, so to lose clarity. Some other students spent much effort in reporting all the computations and failed the request of highlight the solving strategy. As they goes on in using the module and the opened-tasks, they deliver answers more appropriate. As consequence, the discussions between tutor and learner become shorter.

Moreover, we have made comparison among different learning resources presented in IWT. The following figure shows the students preferences.

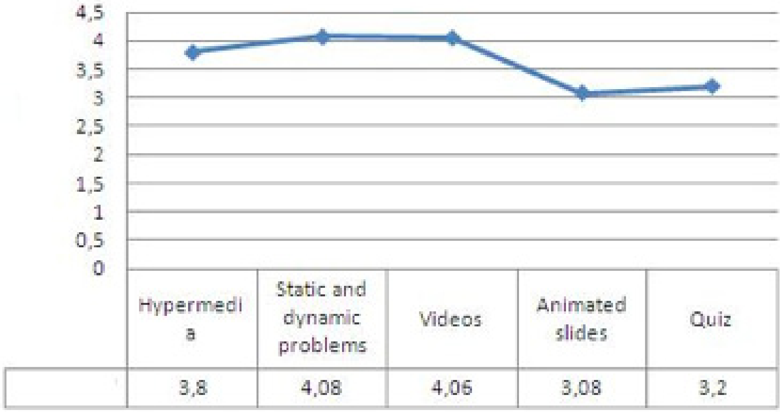

Fig. 5 - Learning resources satisfaction rating

Dynamic problems (that are webM-LAs) seem to be among the most appreciated learning resource. This can be ascribed to more causes and it does not give any indication on the effectiveness of the webM-LAs. Anyway it support Sfard's approach in revaluing technical abilities and in using them to help students in the transition from operational to structural thinking. All this gives good flavour to go further in the research.

Future work

The solution proposed for MatematicaFacile.it and built through the integration of the computing environment Mathematica and the learning platform IWT, allows to combine in an innovative way technologies with learning models. In such a way the attentions is posed not only on the contents but also to the users and their learning needs.

A first step in the direction of future work is to consider the e-learning module and the open-ended tasks in order to implement the Dubinsky pedagogical approach (Asiala *et al.*, 1996). In fact, in the frame of APOS theory, we can consider the ACE Teaching Cycle, constituted by the three components: activities, class discussion and exercises.

We can consider the e-learning module as the first component. Note that the structure of the WM-LA allows to move back and forth from action to process and from process to object. The need of reflecting and interiorizing many repeatable actions to foster the transition from action to process is satisfied by the *quasi-infinity* instances of a WM-LA that the underpinning algorithms are able to generate. On the other hand, after the learner has acquired a process as object, he/she can use it for further manipulation in more sophisticated WM-LA.

The open-ended tasks can be considered as the second component. At the present, we have used it as discussion between student and tutor, but there are no limits to use them in a collaborative setting, adding communication tools.

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